(12)

[REPORT DOCUM]ENTATION PAGE

Form Approved
OPM No. 0704-0188

1. AGENCY USE ONLY *(Leave Blank)*

2. REPORT DATE
December 1992

3. REPORT TYPE AND DATES COVERED
Final

4. TITLE AND SUBTITLE
Elementary and Advanced Computer Projects for the Physics Classroom and Laboratory

5. FUNDING NUMBERS
C - N00014-91-C-0002
PE - 65154N
PR - R0148

6. AUTHOR(S)
J.J. Molitoris

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Center for Naval Analyses
4401 Ford Avenue
Alexandria, Virginia 22302-0268

8. PERFORMING ORGANIZATION REPORT NUMBER
PP 532

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Cleared for Public Release; Distribution Unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

A series of computer based student research projects and programs for use in the physics classroom and laboratory is described.

The commercial programs are SPF/PC, MS Word, T3, Symphony, Mathematica, and FORTRAN. The authors' programs assist data analysis in particular laboratory experiments and make use of the Monte Carlo and other numerical techniques in computer simulation and instruction. The one dimensional Ising model is presented as a good example of the application of both analytical and simulation techniques in physics. The consistent use of this project based teaching approach in a college physics program enhances the students' education and interest.

~~93 3 8 143~~

14. SUBJECT TERMS
Computer aided instruction, Computer applications, Computer programs, Computerized simulations, Education, Physics, Physics laboratories

15. NUMBER OF PAGES
30

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT
CPR

18. SECURITY CLASSIFICATION OF THIS PAGE
CPR

19. SECURITY CLASSIFICATION OF ABSTRACT
CPR

20. LIMITATION OF ABSTRACT
SAR

Professional Paper 532 / December 1992

Elementary and Advanced Computer Projects for the Physics Classroom and Laboratory

by

J. J. Molitoris

Professional Paper 532 / December 1992

Elementary and Advanced Computer Projects for the Physics Classroom and Laboratory

by

J. J. Molitoris

Submarine & Antisubmarine Warfare Department

CNA

CENTER FOR NAVAL ANALYSES

4401 Ford Avenue • Post Office Box 16268 • Alexandria, Virginia 22302-0268

Elementary and Advanced Computer Projects for the Physics Classroom and Laboratory

J.J. Molitoris

Center for Naval Analyses
4401 Ford Avenue
Alexandria, VA 22304

Abstract

A series of computer based student research projects and programs for use in the physics classroom and laboratory is described. The commercial programs are SPF/PC, MS Word, T^3, Symphony, Mathematica, and FORTRAN. The authors' programs assist data analysis in particular laboratory experiments and make use of the Monte Carlo and other numerical techniques in computer simulation and instruction. The one dimensional Ising model is presented as a good example of the application of both analytical and simulation techniques in physics. The consistent use of this project based teaching approach in a college physics program enhances the students' education and interest.

An article for the Journal of College Science Teaching.

I. Introduction

Computer programs in general and Computer Assisted Instruction (CAI) in particular have not lived up to their promise in physics education. Part of the reason for this is perhaps resistance from students and physicists who have not grown up with computers. People, like the objects of physics, have a tendency to resist change (inertia). The impact of the computer is perhaps so great as to require essential modifications in teaching style.[1] Physicists can be quite radical in frontier research (where computers are firmly established in a new scientific triad: experiment, theory, and computation), but very conservative in everyday teaching.[2,3]

However, the Personal Computer (PC) has been around since the 1970's and has just become ubiquitous in the 1980's. Small size and low price ensure wide availability. Hence, perhaps the time is ripe in the 1990's and beyond for educators and students to take full advantage of the PC. Indeed, computer literacy is today one of the important outcomes of a quality education. Computers and computer programs are now a common part of the techniques which aim to improve physics teaching.[1-5] The lecture method, where the information passes from the notes of the instructor to the notes of the student without going through the mind of either, is hopefully on the decline.

What is CAI and how can computers be effectively used? Computers can be used to teach information, skills, and even critical or scientific thinking[4] through a series of interactive computer based lessons - for drill and practice, games, tutoring,

problem-solving, and simulations.[5,6] The goal of CAI is obviously not only to teach by drill and practice the physics concepts which a student has missed in class. However, such drill and practice can be useful[7] in certain circumstances. One must select material to teach in any physics course since you can't cover the whole 700 page text in one year (although many professors try to)! Also, a CAI program for pre-medical students could help them prepare for the MCATs and thus give the instructor more freedom to teach a novel and creative course. Likewise, a CAI program for engineering students could help the students prepare for engineering courses, and allow the instructor the same liberty. Further, in cases where the student/faculty ratio is large (as in many first year courses), the computer can provide the one-on-one environment that is more conducive to education.

II. Computer Based Research Projects

One of the best ways to use computers in physics education, and the focus of our discussion here, is as a tool in individual student research projects for simulating both simple and more complicated phenomena.[5] The instructor must be very knowledgeable about all the projects, perhaps having done them himself/herself in advance. The instructor can integrate the projects into a standard course by an appropriate choice of topics and with in-class student presentations or progress reports. The student must interact significantly with the instructor, who guides him/her through the project. The student

needs to spend much time in the library researching the project in the appropriate journals and books. The instructor can provide a list of references to get the student started. In some cases, the student can also do a related laboratory experiment.

Computational physics projects can help to make a course more contemporary by including exciting information about chaos, fractals, relativity, quantum mechanics, and particle physics. They can reduce drudgery and show the new emphasis on numerical rather than analytic solutions to problems.[1,5] Analytic skills can also be enhanced by emphasizing calculus methods and techniques. Computer projects can help to insure that students improve their conceptual understanding, as well as retain essential facts. Through the computer's speed and power, one can connect the simple physics to help the student understand complex physical or engineering problems.

Computer assisted instruction and simulations are not, however, intended to become a substitute for the teacher.[8] They can only supplement the essential human component of education. Administrators (as well as teachers) must have adequate training in computer use (be computer literate) and be supportive of faculty innovation for computers to be effective. Schools and colleges need to have the requisite financial, programming, and technical support for the instructor.

Clearly computers are not a panacea: they are not the answer to all the problems in physics education. But they can be a powerful instrument to reinforce and supplement the lessons of the classroom. A good CAI program or assigned research project

can motivate some students to spend time researching at the library or inspire them to think and ask questions of their peers and instructors.

We are interested specifically in the application of personal computer hardware and software to a high school or college/university physics environment. What is first necessary is a laboratory of computers dedicated for use by physics students. Assuming a class size of ten to twenty students, a minimum of ten computers is needed. (For departments that lack money, a few personal computers is certainly better than none at all.) These computers need not be IBM compatible, but we have chosen that as the standard. In fact, we have used Zenith computers. Machines of the IBM AT type (Z-200 series) based on the 80286 chip or even the 80386 chip (Z-300 series) are preferred over those which use the slower 8088 chip. The computers could all be linked to a laser printer (HP Laserjet Series II) or, as we have done, two or three computers can share a high quality text/graphics printer (Epson EX-800 or FX-850).

Each machine can be outfitted with two 1.2 MB floppy drives for efficient file transfer and backup. An 80287 math coprocessor is useful for speed in numerical computations and increasing the system recalculation speed. A 20 MB hard drive will do, but 80 to 100 MB will be much better in the long run. For example, if one is to do complicated calculations with advanced students, the 800 routine IMSL FORTRAN libraries alone require 20 MB.

Color monitors make it easier to maintain the students'

interest in this age of color graphics, television, and video. VGA graphics with 640 by 480 pixels is the current high resolution standard and is available with the Z-549 and many other video cards.

Such a physics laboratory[5,9] can be started in numerous ways. For the high school/college/university without much money, a single computer with the necessary hardware and software could be purchased by the instructor for less than $3000. If funds are not available from a departmental budget or the administration, often a local foundation (like the Kiwanis Foundation in Allentown) will provide a grant of up to $2000. Grants are also available from many other sources including the military, the National Science Foundation, and organizations like the Pew Charitable Trust.

III. Commercial Programs

There is a plethora of available programs for an IBM compatible personal computer.[10] What is useful to students and professors of physics are a screen manager (e.g. SPF/PC 2.1), word processor (Microsoft Word 4.0), scientific word processor (T^3 2.21), spreadsheet (Lotus Symphony 2.0), symbolic mathematics package (Mathematica 2.0) and a programming language (Ryan McFarland FORTRAN 2.4)

SPF/PC is a Systems Productivity Facility for the PC costing about $100. It allows you to edit ASCII files, execute DOS commands from a command processor, and even define your own user applications. The computer helps us to manage information, and

programs like SPF/PC aid us in managing the computer. Microsoft Word is a well known word processing program and sells for about \$200. The word processor is used by the student and professor to compose, modify, correct, and finally print research papers. It is useful for both local and global revision, the latter being done on a printed hard copy.

T^3 is the scientific word processing system on which this document was prepared (cost \$600). It is essential for the typing of mathematical equations for lab reports, lab write-ups, tests, and research papers. Symphony can be used as a word processor or data base manager, as well as a spreadsheet (cost \$400). We like the look and feel of Symphony as a spreadsheet over that of Lotus 123. The instructor can use Symphony to keep a record of grades. Both the students and the professor can use it to tabulate numbers and plot graphs.

Mathematica is a numeric, symbolic, and graphical system for doing mathematics by computer (cost \$800). It is able to differentiate and integrate symbolically, numerically evaluate a definite integral, numerically solve a differential equation using a 4th order Runge-Kutta method, solve eigenvalue and eigenvector problems, do matrix multiplication, simply plot functions in two or three dimensional contour or perspective and in color, and also perform many other useful tasks.

FORTRAN is still the language of science and engineering in industry and government laboratories (alth. .gh C is becoming a powerful competitor). RM/FORTRAN (cost \$400) is one implementation of this powerful and useful programming language.

It is important that the student learn to program not only as a useful and marketable skill, but also because this helps one to think about how to approach problems in general. Furthermore, the content and operation of one's own program is more manifest and easily adaptable than say a spreadsheet or other commercial program.

IV. A Computer Programming Laboratory

We mention here a computer laboratory[5,9] which has been used successfully at both Muhlenberg College and Cedar Crest College in Allentown with both introductory and advanced students. A copy of the laboratory hand-out is available on request from the author. The laboratory exercise is called Computers and Programming and the students take about two lab periods (4 hours) to complete the activity. This instruction is useful because throughout the students' four year long physics education, computer programs are used to aid the experimental data analysis and for simulations.

The object of the exercise is to study computers and FORTRAN programming. The basic history of computers and their use is discussed as well as the essentials of FORTRAN programming. It is really remarkable that today a personal computer has a mass less than 50 kg (some notebook computers even less than 1 kg), occupies less than 1/2 square meter, and can do more than the 130 Megagram 140 square meter 1946 University of Pennsylvania ENIAC could.

By the end of the laboratory, the student has created a simple

FORTRAN program to (for example) find the average and standard deviation of n numbers. Clearly, the physics major needs to go beyond this introductory laboratory exercise. This can happen through a formal course in computer science, course related projects, and/or a summer research project. Course related computer projects are most useful since they stimulate interest in physics and allow the students a unique way to demonstrate their understanding.

V. CAI and Computer Programs for Student Research Projects

A CAI package of 114 programs in 12 volumes on general physics from vectors to atomic and nuclear physics is available from Cross Educational Software for about $300. These programs feature extensive graphics with animation, color, and sound. A summary of other available software may be found in the literature.[10]

The programs written by the author have been developed and tested for use by first year advanced and also upper class physics students. The basic idea is that the student works on his/her own computer program as part of a research project. The programs and projects described here should thus be a guide to the professor and student. As part of this project based approach to physics eduation, the student should present one or more talks on the project. These oral presentations can be done in class, at a specially scheduled mini-conference, or even at a local physics society meeting (NSTA, AAPT, or APS). If the presentations are to be done in class as part of a course, the

intructor can choose one or two readings from appropriate journals (e.g. Scientific American or Physics Today) or books for the students to read for the class. In this way, all the students can take part in a discussion about the project and the project becomes part of the material studied for the course.

These computer projects have been used in the classroom and laboratory, as well as outside formal class meetings. This naturally requires student interest and commitment. Actually, students enjoy the projects and programs and consider them important parts of their physics education. The programs are especially relevant for a course in Classical Mechanics, Nuclear Physics, or Statistical Physics and as supplements to other courses. The excellent graphics enhance the learning of material and retain the students' interest. The organization of the programs are logical and apparent to the user. All responses to program questions have an indicated default value.

The Monte Carlo program *monte* (development initiated by Dr. Pat Bronson) begins by spelling out the words Monte Carlo on the screen using the BASIC random number generator. After a title page comes some general information about the Monte Carlo method and probability theory.[11,12] At this point, one is presented with a menu which allows s/he to quit the program, obtain further information about the Monte Carlo technique, learn about the random walk, nuclear decay, the absorption of pions, or scattering processes. There are numerous references in the literature to the Monte Carlo method.[13-17]

The Math Tutor program *math* (development initiated by Chris

Brueningsen) allows students the opportunity to be introduced to and/or review scientific notation, basic algebra, trigonometry, and the calculus. The physics major needs to go beyond this and learn about methods of numerical integration, random numbers, and how to numerically solve differential equations. Programs *numint* and *numode* facilitate this study.

Other computer software available to aid the data analysis of laboratory experiments[9] include: *vector* and *sumxy* for a force table experiment; *kinem* for a linear air track experiment; *plotit*, *linzer*, *quazer*, *linlst*, and *qualst* for any experiment which requires plotting data or linear or quadratic least squares fits (zero-intercept or regular); *shm* for a pendulum simple harmonic motion experiment; and *consmom* for a planar air table experiment.

Most of these programs are written in FORTRAN and require EGA graphics and a numerical coprocessor. Output may be directed to the screen, an Epson compatible printer, a Laserjet compatible printer, or a Hewlett-Packard 7475 plotter. The quadratic least squares program *qualst* is part of the 1990 Plot88 shareware collection. EGA/VGA graphics from any BASIC program or Mathematica plot may be dumped to the HP 7475 plotter using a program called *markplot* developed by a computer science student (Mark Hyatt) at Muhlenberg College.

Some classical mechanics programs are: *lissaj* which draws Lissajous figures; *pendorb* which draws phase space orbits for a pendulum; *shmorb* which draws phase space orbits for a mass on a spring in simple harmonic motion; *realfall* which solves the

equations of motion for a spherical object falling in a fluid medium using 4th order Runge-Kutta methods; and *real2d* which does the same thing but for two dimensional projectile motion. The program *chaos* generates time series and the bifurcation diagram for the logistic difference equation, as well as allowing an investigation of the dynamics of the damped forced nonlinear pendulum. *Mandel* and *mandplt* allow an investigation of the Mandelbrot set and *koch* studies the Koch fractal.

For electricity and magnetism, the program *electric* allows one to plot electric field vectors for n point charges. *Equipot* plots the equipotential lines for a single point charge and *equidip* plots the equipotential lines for a dipole.

The program *random* allows an investigation of the randomness of the pseudo-random numbers generated by an IMSL FORTRAN routine and some other algorithms. *Ranwalk* implements the one to three dimensional random walk using the Monte Carlo method. *Isingcan* allows the Monte Carlo study of the one dimensional Ising model with regard to the total energy and the magnetization. Comparison is made to the complicated exact theoretical formulae in both *isingcan* and *ranwalk*.

Several other programs are also available free on request from the author. *Impact* studies the material impact of a right circular metal cylinder on an anvil. Computer software related to nuclear physics - in particular the Fermi gas model of the ground and excited states, giant quadrupole resonances, and a shock model of central nucleus-nucleus collisions - are *ftfermcm*,

quad, and *shockcm*. Clearly, there are unlimited possibilities for student research projects.

VI. Some Sample Results and The Ising Model

Let us consider some of the graphical results of the student research projects and computer programs. On the computer, all of the figures would be displayed in color. Figure 1 shows a Lissajous figure from program *lissaj*, which occurs in the study of two dimensional oscillations in a Classical Mechanics course. The curve is closed because the motion repeats itself at regular intervals of time. Figure 2 shows a stage in the development of the Koch fractal from program *koch*. This curve has a fractal or fractional dimension of 1.26, i.e. it is between a line and a plane in dimensionality.

Figure 3 shows a chaotic time series from program *chaos*. Here, one studies the complex, irregular, and apparently random behaviour of many physical phenomena. The values of x in this discrete sequence for times 0,1,2,... have no obvious pattern. Figure 4 shows the velocity-time curve for the free fall case (straight line) and the realistic fall of a .05 cm steel sphere falling in air, as simulated with program *realfall*. Notice that the object achieves terminal speed after about 2 seconds. Figure 5 shows the probability of being at a particular distance r from the origin in a three dimensional three step random walk, from program *ranwalk*. We take fixed steps of unit size with the angles θ and ϕ chosen randomly. The computer simulation

probability (boxes) and the analytic theory (solid line) agree very well.

To be more specific, we now consider the Ising model simulation in more detail. A more mathematical discussion of this and some of the other research projects is available on request.[18-21] Many of these projects are discussed in a number of computer simulation and modeling books[22-25] but not always in sufficient detail. The Ising model attempts to simulate the structure of a ferromagnet.

The system considered is an array of N lattice sites or spins at temperature T. Each lattice site has an associated spin $\sigma_j = \pm 1$. The Hamiltonian or total energy for the n dimensional Ising model involves an interaction strength a, the single particle magnetic moment μ, and the external magnetic field B. The actual value of a depends upon more detailed theory. We consider here the one dimensional Ising model (n = 1).

The analytic formulae for the energy U = U(B,T) and magnetization M = M(B,T) can be obtained exactly using statistical physics and are quite complicated.[18] One would certainly not be able to guess them from any data or simulation. However, the analytic theory is essential and in the one dimensional case tractable.

Program *isingcan* implements the one dimensional Ising model discussed above using the Monte Carlo method. The program is available free of charge on request from the author. The user inputs the size of the ring or the number of spins N, the temperature T, and the magnetic field B to be studied. The

program then either loops over a range of B values at fixed T or a range of T values at fixed B, as chosen by the user. The number of Monte Carlo steps n_m and number of initial configurations n_c are also input.

An IMSL random number subroutine is used to set $\sigma_i = +1$ if a uniform [0,1] random number is greater than 0.5 and -1 otherwise. This first ensemble or configuration of spins is then run through the n_m Monte Carlo steps.

In each step, a spin (say j) is randomly selected and the change in energy from flipping that spin computed. If the energy change is less than or equal to zero, then the spin is flipped. The Monte Carlo approach aims to minimize the energy or relax the system towards thermal equilibrium. Otherwise a temperature dependent Boltzmann factor is computed. If a random number is less than or equal to this factor, then the spin is flipped. This procedure is repeated N times.

The energy and magnetization after relaxation are saved after the n_m Monte Carlo steps. Then the procedure is repeated for a new ensemble or initial configuration.

Shown in Figure 6 is a comparison between the computer simulation (boxes) and the analytic theory (solid line) for B = 1 and a range of temperatures. The lattice size or number of spins is 100. At zero temperature, all of the spins tend to align with the magnetic field, as one would expect. For higher temperatures, the thermal energy of the system tends to destroy the alignment. The agreement of the simulation with the exact results is quite remarkable.[18]

VII. Summary and Conclusions

Typically, an advanced student would work on a research topic like the Ising model for a semester or more. Such research projects have strengthened the physics major at Muhlenberg College. Over the past three years about 90% of the graduating physics majors have worked on Monte Carlo and other computer projects. For the past three summers, 50% of our majors were involved in a for pay Summer Science Program at Muhlenberg utilizing computational research projects.

This system of programs remains under development. It will be extended and modified by the author and his students over time. Any of the topics in the *chaos*, *koch*, *monte*, *isingcan*, *quad*, *random*, *ranwalk*, *realfall*, *real2d*, and other programs make excellent review and research topics for the physics student.

As we have seen, computer based student research projects in general have a vital role as a powerful innovative teaching tool. This approach to physics education is based on the premise that students benefit from well-structured material, graphic presentation, and supplementary ideas and instruction. Through simulation programs and the laboratory interfacing of computers, students can learn through analyzing, exploring, and manipulating their environment.

Our thanks to the many students who have worked on these programs including Robin Balla, Dave Brown, Scott Kaeppel, Rory Klinger, Jeff Mahn, Joe Maselli, Kathy Massopust, Brennan Milligan, Drew Paulson, Chris Scott, and Matt Wingate. We are also grateful to Barbara Negus of the Center for Naval Analyses

for her help in producing this preprint, John Hrizuk of the Muhlenberg College Academic Computing Center for countless advice and discussion of software and hardware, and Steve Daniels / Frank Moscatelli for their direction of the Physics Pew Subprogram. I thank the Center for Naval Analyses, especially the Submarine Department, for its continuing support of my research endeavors. Funding from the Kiwanis Foundation of Allentown, Muhlenberg College, the Pew Charitable Trust, and Universal Energy Systems is gratefully acknowledged.

References

1. J. Kemeny, How Computers Have Changed the Way I Teach, Academic Computing, May/June 1988, page 44.

2. E. Redish, From Here to the Future: The Impact of the Computer on College Physics Teaching, Academic Computing, Nov. 1988, page 18.

3. A. Bork, Computer Based Instruction in Physics, Physics Today, Sept. 1981, page 24.
D. Hamann, Computers in Physics: An Overview, Physics Today, May 1983, page 25.
L. Kadanoff, Interactive Computation for Undergraduates, Physics Today, Dec. 1988, page 9.

4. J. Larkin and R. Chabay, chapter 8 in Toward the Thinking Curriculum, 1989 Association for Supervision and Curriculum Development Yearbook.

5. J. Molitoris, Elementary and Advanced Computer Use in an Undergraduate Physics Program, Proceedings of the Bucknell University Conference on Undergraduate Physics Education, Swarthmore(PA): Pew Physics Program, 1991.

6. A. Collette and E. Chiappetta, Science Instruction, Columbus (OH): Merrill, 1986.

7. J. Weyh and J. Crook, CAI Drill and Practice, Academic Computing, May/June 1988, page 32.

8. H. Brady, Classroom Computer Learning, Nov./Dec. 1985, page 24.

9. J. Molitoris, R. Milligan, W. Loy, R. Boyer, H. Raub, Laboratory Experiments for the 21st Century, Allentown: Muhlenberg College, 1988.

10. M. Gjertsen and J. Risley, Educational Software for Teaching Physics, The Physics Teacher, May 1987, page 301.

11. Lady Luck, W. Weaver, NY: Dover, 1987.

12. M. Kac, Probability, Scientific American, Sept. 1964, page 92.
A. Ayer, Chance, Scientific American, Oct. 1965.

13. A. Howie, Monte Carlo Method, page 184, Encyclopedic Dictionary of Physics, ed. J. Thewlis, NY: Pergamon, 1966.
I. Good, Monte Carlo Method, page 383, Encyclopedia of Science and Technology, ed. S. Parker, NY: McGraw Hill, 1987.

14. C. Whitney, Casino Physics in the Classroom, Am. J. Phys., 54 (1986) page 1079.

15. P. Bak, Doing Physics With Microcomputers, Physics Today, Dec. 1983, page 25.

16. D. Raeside, An Introduction to Monte Carlo Methods, Am. J. Phys. 42 (1974) page 20.

17. D. Landau and R. Alben, Monte Carlo Calculations as an Aid in Teaching Statistical Mechanics, Am. J. Phys. 41 (1973) page 394.

18. J. Molitoris, The One Dimensional Ising Model in Mathematical Physics, Swarthmore(PA): Pew Physics Program, 1990.

19. J. Molitoris, The One, Two, and Three Dimensional Random Walk in Mathematical Physics, Swarthmore(PA): Pew Physics Program, 1990.

20. J. Molitoris, Models of Projectile Impact, Swarthmore(PA): Pew Physics Program, 1989.

21. J. Molitoris, A Monte Carlo Study of the Giant Quadrupole Resonance, Allentown(PA): Muhlenberg College, 1991.

22. H. Gould and J. Tobochnik, <u>Computer Simulation Methods</u>, vol. 2, page 511, NY: Addison-Wesley, 1988.

23. S. Koonin and D. Meredith, <u>Computational Physics</u>, NY: Addison-Wesley, 1990.

24. W. Cheney and D. Kincaid, <u>Numerical Mathematics and Computing</u>, 2nd ed., Pacific Grove(CA): Brooks/Cole, 1985.

25. W. Press, et. al., <u>Numerical Recipes</u>, NY: Cambridge University Press, 1989.

Appendix: List of Manufacturers

Cross Educational Software
1802 N. Trenton St.
P.O. Box 1536
Ruston, LA 71270
318 255 8921

IMSL Sfun/Math/Stat Libraries
IMSL, 2500 ParkWest Tower One
2500 CityWest Blvd.
Houston, TX 77042
713 782 6060

Mathematica 2.0
Wolfram Research Inc.
P.O. Box 6059
Champaign, IL 61821
217 398 6500

Plot88 rev. 19
Plotworks, Inc.
16440 Eagles Crest Road
Ramona, CA 92065
619 457 5090

Ryan McFarland/FORTRAN 2.4
Austec, Inc.
609 Deep Valley Drive
Rolling Hills Estates, CA 90274
213 541 4828

Symphony 2.0
Lotus Development Corporation
55 Cambridge Parkway
Cambridge, MA 02142
617 253 9150

SPF/PC 2.1
Command Technology Corporation
1040 Marina Village Parkway
Alameda, CA 94501
510 521 5900

T^3 2.21
TCI Software Research Inc.
1190-B Foster Road
Las Cruces, NM 88001
800 874 2383

Figures

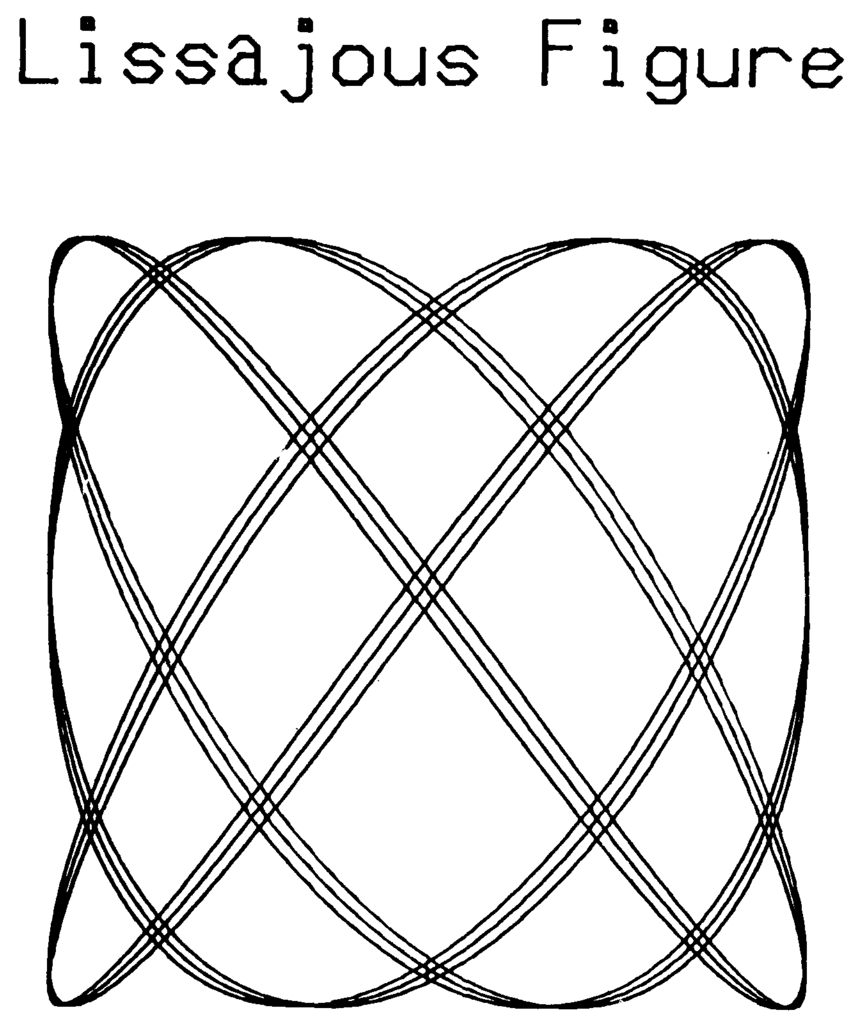


Figure 1: A Lissajous figure for two dimensional oscillations.

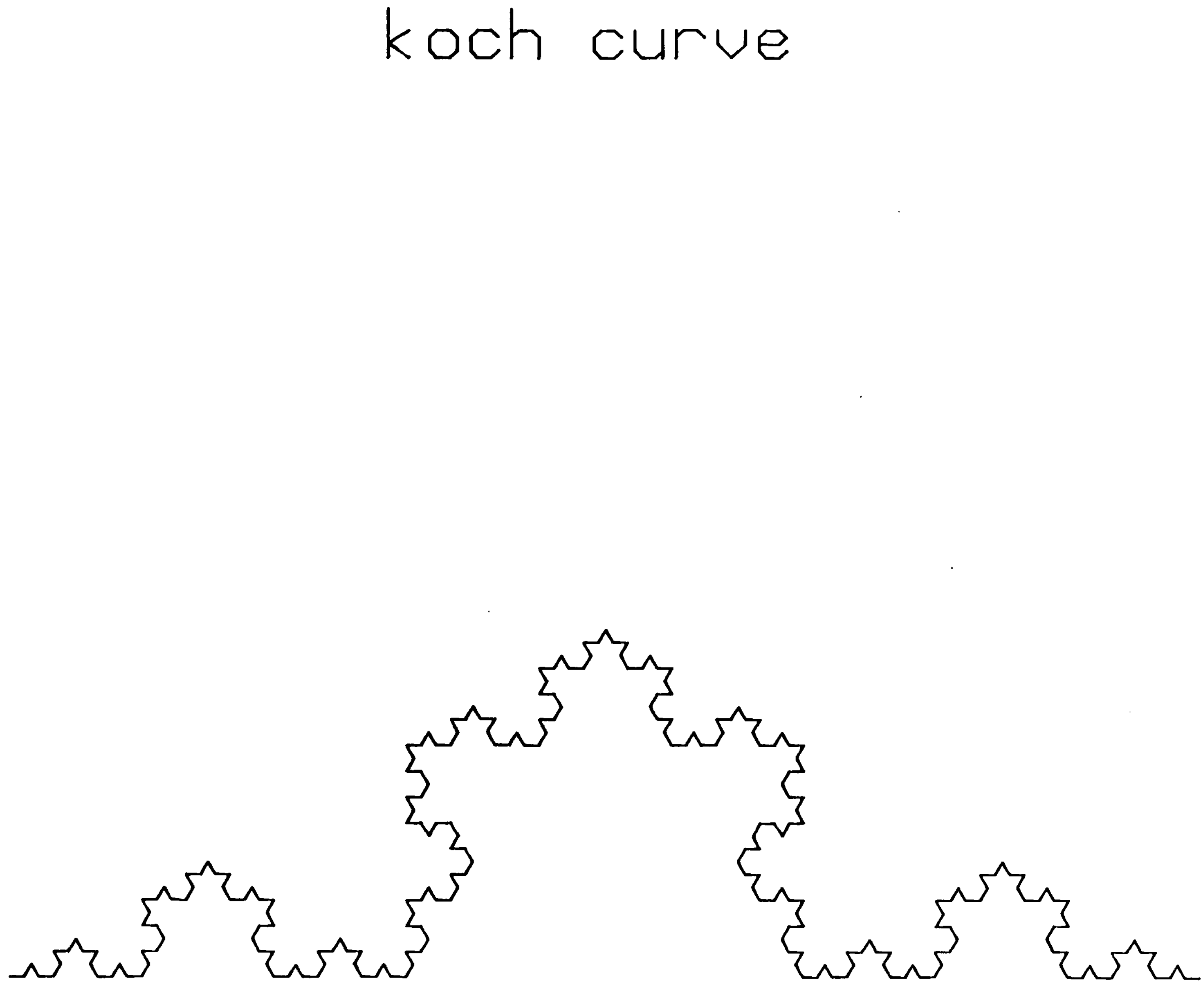


Figure 2: The Koch fractal with fractional dimension 1.26.

Figure 3: A chaotic time series from the logistic difference equation.

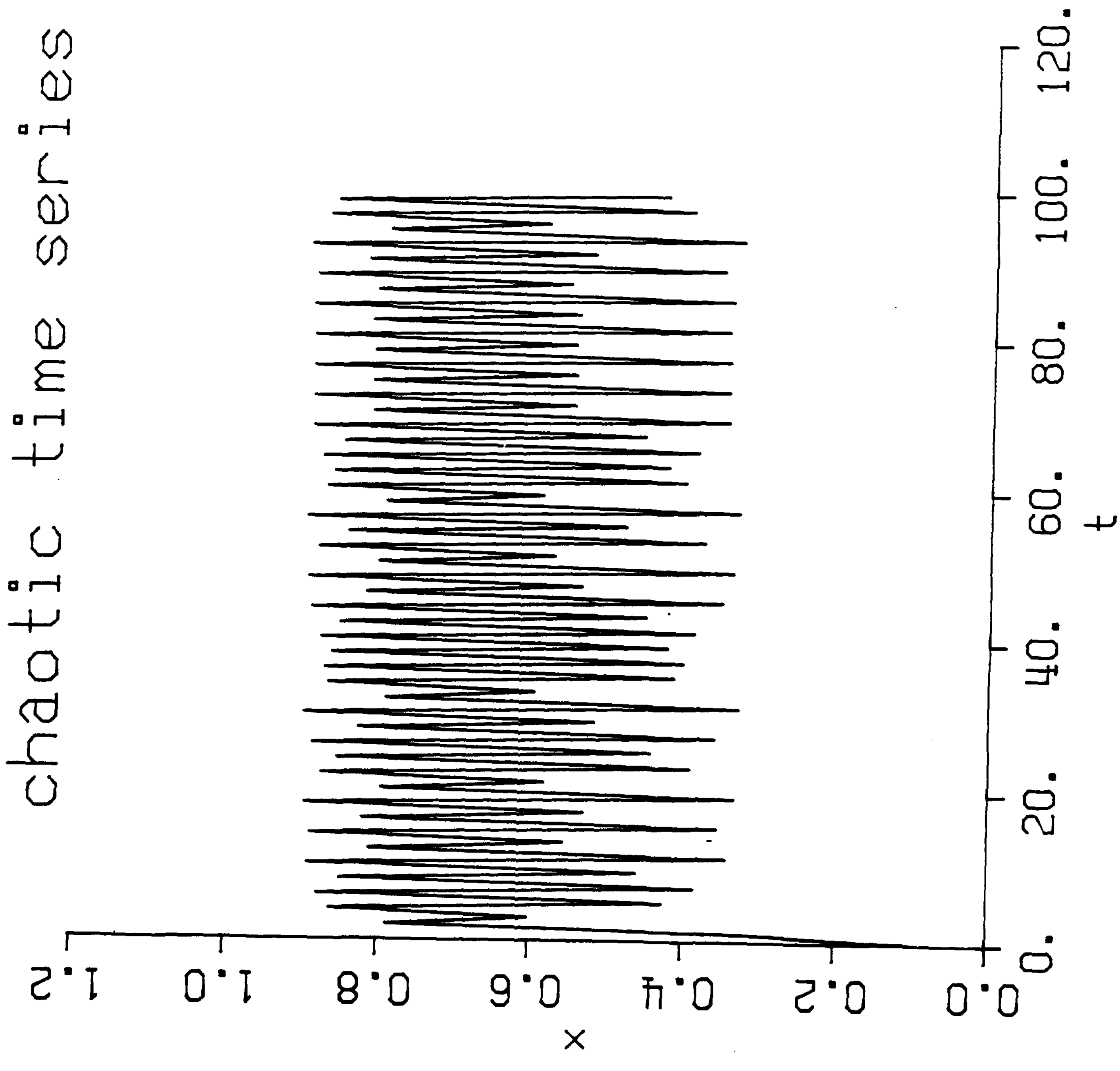


Figure 4: The velocity versus time for vacuum and realistic fall.

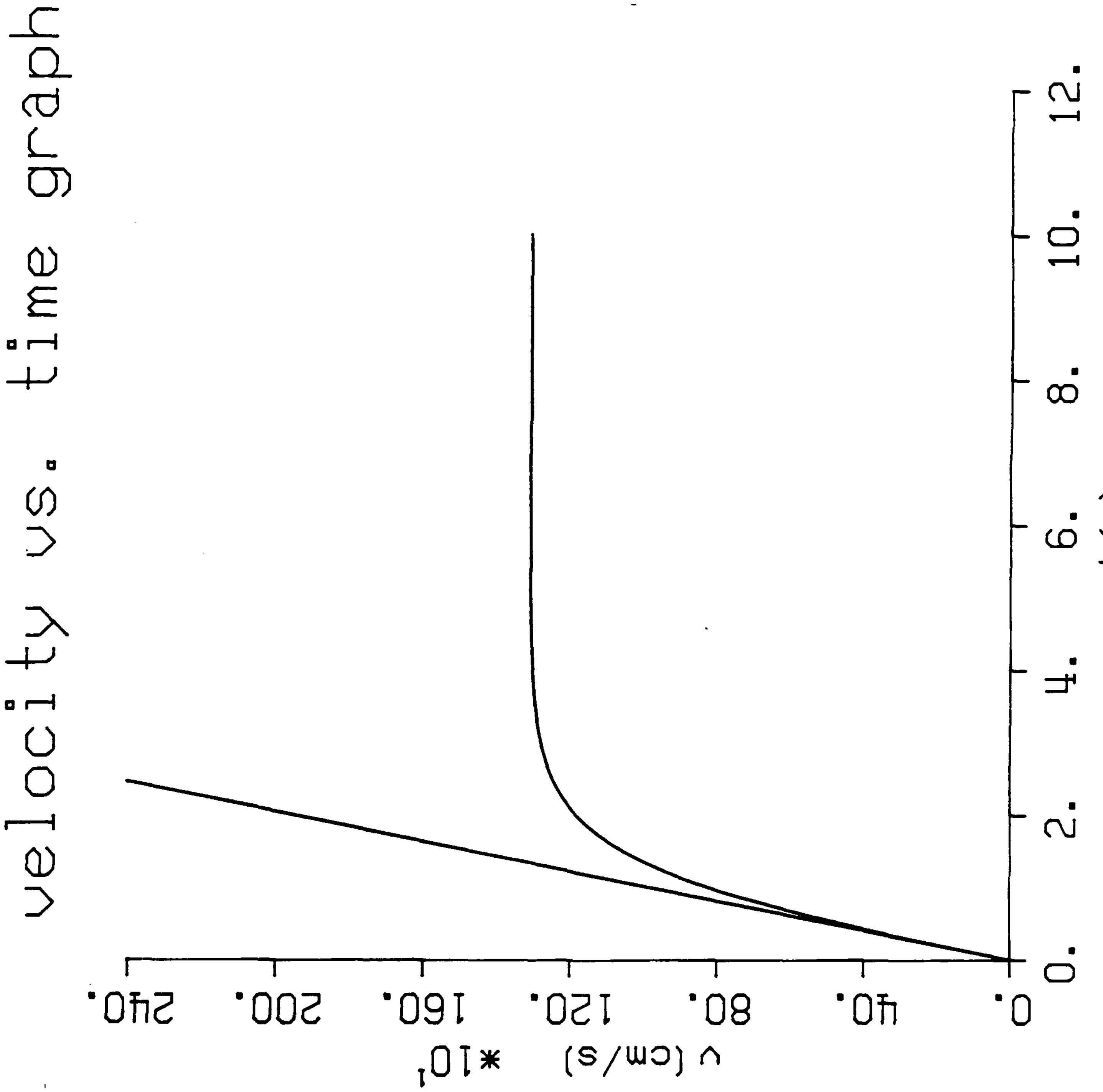


Figure 5: The three step three dimensional random walk probability function.

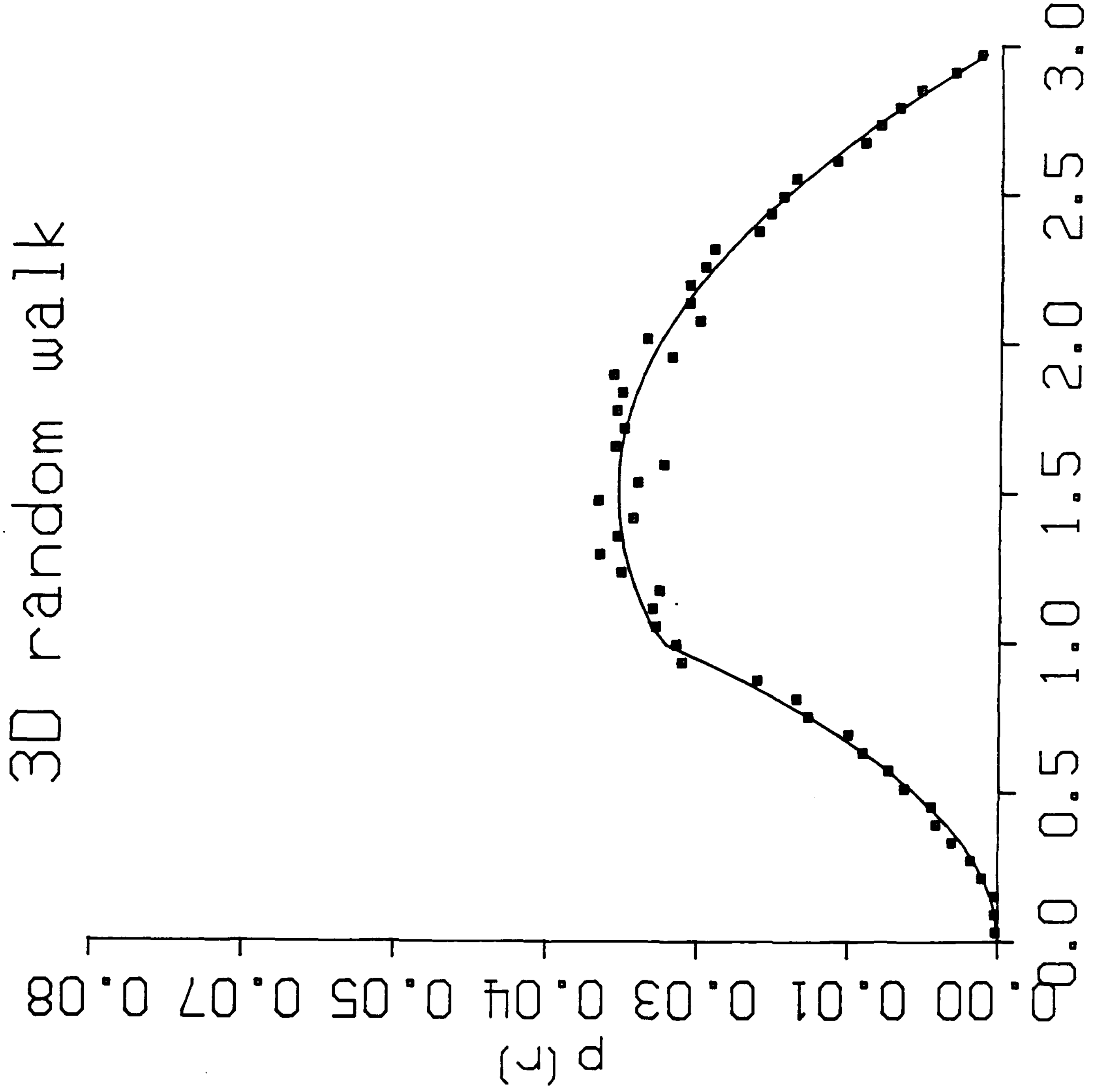
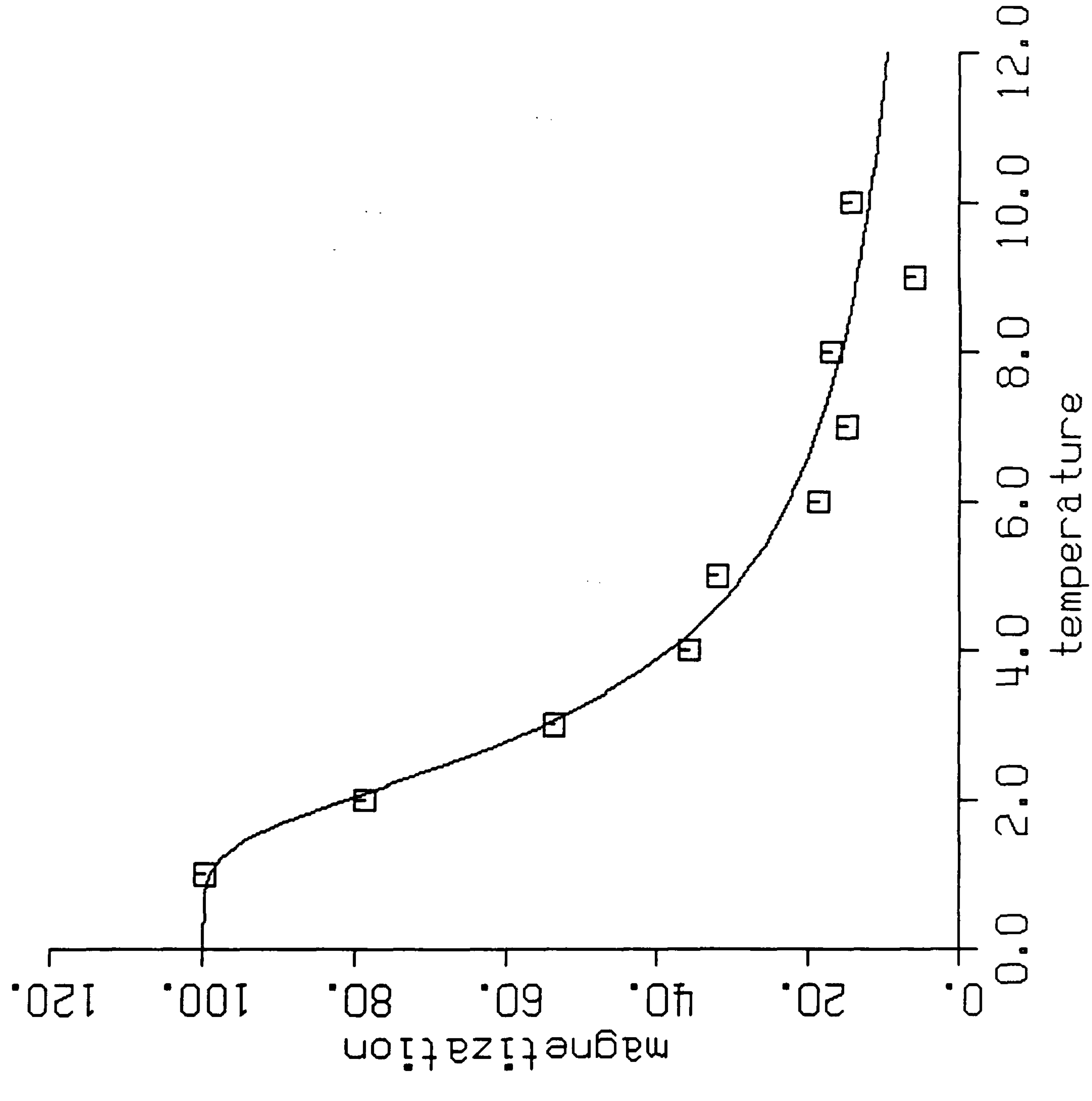


Figure 6: The magnetization versus temperature in the one dimensional Ising model.



CNA PROFESSIONAL PAPER INDEX[1]

PP 407[2]
Laird, Robbin F., *The French Strategic Dilemma*, 22 pp., Nov 1984

PP 415
Mizrahi, Maurice M., *Can Authoritative Studies Be Trusted?* 2 pp., Jun 1984

PP 416
Jondrow, James M. and Levy, Robert A., *The Displacement of Local Spending for Pollution Control by Federal Construction Grants*, 6 pp., Jun 1984 (Reprinted from *American Economic Review*, May 1984)

PP 418
Reslock, Patricia A., *The Care and Feeding of Magnetic Tapes*, 7 pp., Jul 1984

PP 420
Weiss, Kenneth G., *The War for the Falklands: A Chronology*, 32 pp., Aug 1982

PP 422
Quester, Aline and Marcus, Alan, *An Evaluation of the Effectiveness of Classroom and On the Job Training*, 35 pp., Dec 1984. (Presented at the Symposium on Training Effectiveness, NATO Defense Research Group, Brussels, 7-9 January 1985)

PP 423
Dismukes, N. Bradford and Weiss, Kenneth G., *MARE MOSSO: The Mediterranean Theater*, 26 pp., Nov 1984. (Presented at the Seapower Conference, Washington, D.C., 26-27 November 1984)

PP 424
Berg, Dr. Robert M., *The CNA Ordnance Programming Model and Methodology*, 27 pp., Oct 1984. (Presented at the ORSA-MAS/MDRS Symposium, Washington, Aug 1984)

PP 425
Horowitz, Stanley A. and Angier, Bruce N., *Costs and Benefits of Training and Experience*, 18 pp., Jan 1984. (Presented at the Symposium on Training Effectiveness, NATO Defense Research Group, Brussels, 7-9 January 1985)

PP 427
Cavalluzzo, Linda C., *OpTempo and Training Effectiveness*, 19 pp., Dec 1984. (Presented at the Symposium on Training Effectiveness, NATO Defense Research Group, Brussels, 7-9 January 1985)

PP 428
Matthes, Greg, Cdr., USN and Evanovich, Peter, *Force Levels, Readiness, and Capability*, 24 pp., Nov 1984. (Presented at the ORSA-TIMS 26-28 November Meeting, Washington, D.C.)

PP 429
Perla, Peter P. and Barret, Raymond T., LCdr., USN, *Wargaming and Its Uses*, 13 pp., Nov 1984. (Published in the *Naval War College Review*, XXXVIII, No. 5 / Sequence 311, September-October 1985)

PP 430
Goldberg, Matthew S., *The Relationship Between Material Failures and Flight Hours: Statistical Considerations*, 18 pp., Jan 1985

PP 431
McConnell, James M., *A Possible Change in Soviet Views on the Prospects for Antisubmarine Warfare*, 19 pp., Jan 1985

PP 432
Marcus, Alan J. and Curran, Lawrence E., Cdr., USN, *The Use of Flight Simulators in Measuring and Improving Training Effectiveness*, 29 pp., Jan 1985. (Presented at the Symposium on Training Effectiveness, NATO Defense Research Group, Brussels, 7-9 January 1985)

PP 433
Quester, Aline O. and Lockman, Robert F., *The All Volunteer Force: Outlook for the Eighties and Nineties*, 20 pp., Mar 1984. (To be published in *Armed Forces and Society*, 1985)

PP 435
Levine, Daniel B. and Jondrow, James M., *Readiness or Resources: Which Comes First?* 12 pp., Mar 1985

---

1. CNA Professional Papers with an AD number may be obtained from the National Technical Information Service, U.S. Department of Commerce, Springfield, Virginia, 22151. Other papers are available from the Management Information Office, Center for Naval Analyses, 4401 Ford Avenue, Alexandria, Virginia, 22302-0268. An index of selected publications is also available on request. The index includes a listing of professional papers, with abstracts, issued from 1969 to December 1983.
2. Listings for Professional Papers issued prior to PP 407 can be found in *Index of Selected Publications* (through December 1983), Mar 1984.

PP 436

Goldberg, Matthew S., *Logit Specification Tests Using Grouped Data*, 26 pp., Jan 1985

PP 438

Fletcher, Jean W., *Supply Problems in the Naval Reserve*, 14 pp., Feb 1986. (Presented at the Third Annual Mobilization Conference, Industrial College of the Armed Forces, National Defense University)

PP 440

Bell, Jr., Thomas D., *The Center for Naval Analyses Past, Present, and Future*, 12 pp., Aug 1985

PP 441

Schneiter, George R., *Implications of the Strategic Defense Initiative for the ABM Treaty*, 13 pp., Feb 1986. (Published in Survival, September/October 1985)

PP 442

Berg, Robert; Dennis, Richard; and Jondrow, James, *Price Analysis and the Effects of Competition*, 23 pp., Sep 1985. (Presented at the Association for Public Policy Analysis and Management—The Annual Research Conference, Shoreham Hotel, Washingtong, D.C., 25 October 1985)

PP 443

FitzGerald, Mary C., *Marshal Ogarkov on Modern War: 1977-1985*, 65 pp., Mar 1986

PP 445

Kober, Stanley, *Strategic Defense, Deterrence, and Arms Control*, 23 pp., Aug 1986. (Published in *The Washington Quarterly*, Winter 1986)

PP 446

Mayberry, Paul W. and Maier, Milton H., *Towards Justifying Enlistment Standards: Linking Input Characteristics to Job Performance*, 11 pp., Oct 1986. (Paper to be presented at 1986 American Psychological Association Symposium entitled "Setting Standards in Performance Measurement.")

PP 448

Cymrot, Donald J., *Military Retirement and Social Security: A Comparative Analysis*, 22 pp., Oct 1986

PP 449

Richardson, Henry R., *Search Theory*, 13 pp., Apr 1986

PP 450

Perla, Peter P., *Design, Development, and Play of Navy Wargames*, 32 pp., Mar 1987

PP 451

FitzGerald, Mary C., *The Soviet Leadership on Nuclear War*, 40 pp., Apr 1987

PP 452

Mayberry, Paul W., *Issues in the Development of a Competency Scale: Implications for Linking Job Performance and Aptitude*, 22 pp., Apr 1987

PP 453

Dismukes, N. Bradford, *Strategic ASW and the Conventional Defense of Europe*, 26 pp., Apr 1987

PP 454

Maier, Milton, *Marine Corps Project to Validate the ASVAB Against Job Performance*, 14 pp., May 1987

PP 455

Bennett, Allan, *Continuous Dependence on Modeling in the Cauchy Problem for Nonlinear Elliptic Equations*, 49 pp., Apr 1987

PP 456

Gates, Stephen, *Simulation and Analysis of Flight Deck Operations on an LHA*, 81 pp., Jun 1987

PP 457

Walne, George N., *AFP 110-31: The Conduct of Armed Conflict and Air Operations and the Linebacker Bombing Campaigns of the Vietnam War*, 36 pp., Nov 1987

PP 458

Walt, Stelphen M., *Analysts in War and Peace: MccGwire, McConnell, and Admiral Gorshkow*, 63 pp., Sep 1987

PP 459

Hodes, P. W., *Simulating Cable Logging in the Allegheny Region of Pennsylvania: A Case Study*, 84 pp., Jul 1987

PP 460

FitzGerald, Mary C., *Arms Control and the New Revolution in Soviet Military Affairs*, 20 pp., Aug 1987

PP 461

FitzGerald, Mary C., *The Soviet Military on SDI*, 39 pp., Aug 1987

PP 465

Mayberry, Paul W., *Validity Results for the Marine Corps Job Performance Measurement Project*, 7 pp., Nov 1989

PP 469

Patrick, Meredith W., Capt., USN (Ret); Tindle, John, Cdr., USN; and Caldwell, Steven R., LCdr., USN, *Definitions of Key Terms for Conventional Strike Warfare and Antisurface Warfare*, 36 pp., April 1990

PP 470

Brandt, Linda S., *Statement by Dr. Linda S. Brandt, Center for Naval Analyses, Before the Subcommittee on Industry and Technology, Senate Armed Services Committee*, 36 pp., Nov 1989

PP 471

Holliday, Mary Robin, *Methodology of an Event-Driven Monte Carlo Missile Simulation*, 6 pp., Nov 1989. (As presented to the Seventh International Conference on Mathematical and Computer Modeling, 3 Aug 1989)

PP 472

Siegel, Adam B., *The Wartime Diversion of U.S. Navy Forces in Response to Public Demands for Augmented Coastal Defense*, 49 pp., Nov 1989

PP 477

Eskew, Henry L., *Cost Minimization and Uncertainty in Defense Manufacturing*, 12 pp., Mar 1990

PP 481

Cox, Gregory V., *A Redundant Linear Constraint Elimination Technique*, 18 pp., Apr 1990

PP 488

Van Metre, Lauren L., *Soviet Style Conversion: Will It Succeed?*, 36 pp., Jul 1990

PP 492

Mayberry, Paul F., *Evaluating Minimum Aptitude Standards for the Infantry Occupational Field*, 20 pp., Aug 1990

PP 496

Cobb, Tyrus W., *Collective Security in the Post Cold-War Period*, 24 pp., Nov 1990

PP 499

Siegel, Adam B., *A Brave New Curriculum for a Brave New World?*, 24 pp., March 1991

PP 502

Oklhovsky, Paul, *The Greek Civil War: An Examination of America's First Cold War Victory*, 18 pp., Dec 1990

PP 524

Eskew, Henry L., *Aircraft Cost and Development Program Length: An Empirical Analysis*, 29 pp., Dec 1992

PP 525

Thompson, Frederick D., *A Three-Ocean Joint Navy*, 10 pp., Nov 1991

PP 531

Cooke, Timothy W. and Jondrow, James M., *Costs of Improving Recruit Aptitudes: A Joint Product Approach*, 48 pp., Nov 1992

PP 532

Molitoris, J.J., *Elementary and Advanced Computer Projects for the Physics Classroom and Laboratory*, 30 pp., Dec 1992

PP 533

Tighe, Carla E. and Michener, Ron, *A Poisson Regression Model of Highway Fatalities*, 7 pp., Dec 1992